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Vacuum Laser Acceleration

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Vacuum Laser Acceleration

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The purpose of this communication is to comment on and discuss laser acceleration of electrons in vacuum. In particular, we will: i) critique the recent paper by C. M. Haaland[1], titled "Laser Electron Acceleration in Vacuum," ii) discuss some general features and characteristics of laser acceleration in vacuum, and iii) propose a vacuum laser acceleration concept called the "vacuum beat wave accelerator".

In Ref. 1 a vacuum laser acceleration configuration was proposed and analyzed. In this scheme a pair of linearly-polarized laser beams with Gaussian profiles, having the same frequency, are focused and intersected in vacuum, see Fig. 1. Here, the first laser propagates along the z_1-axis and the second laser propagates along the z_2-axis, where the z_1-axis and the z_2-axis are rotated by the angles of θ and $-\theta$, respectively, with respect to the z-axis. The phases of the lasers are such that the transverse electric fields cancel along the axis while the axial fields add. Properly phased electrons injected along the z-axis can be trapped and accelerated by the net axial component of the laser field. The analysis of this configuration contained in Ref. 1 is flawed for a number of reasons, e.g., the vacuum fields are physically unrealizable. Reference 1 arrives at the incorrect conclusion that "the electron will acquire a net energy gain in this scheme when the laser-electron interaction is integrated along the z-axis from minus to plus infinity". We find, on the other hand, that the net energy gain vanishes over an infinite interaction

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region, which is in agreement with the Lawson-Woodward[2-5] acceleration theorem.

The axial electric field associated with the intersecting laser beams can be calculated by writing the laser fields in the (x,y,z) coordinate frame. In the (x_1,y_1,z_1) coordinate frame, where $x_1 = x\cos\theta - z\sin\theta$, $y_1 = y$, and $z_1 = z\cos\theta + x\sin\theta$, the electric field of the first laser beam consists of a transverse and axial component, $\underset{\sim}{E}_1(x_1,y_1,z_1,t) = E_{x1}\hat{e}_{x1} + E_{z1}\hat{e}_{z1}$ where $\hat{e}_{x1}$ and $\hat{e}_{z1}$ are unit vectors. In the paraxial approximation ($\lambda \ll w_o$), the electric field components of a Gaussian laser beam are given by

$$E_{x1} = E_{o1}\frac{w_o}{w_1}\exp(-r_1^2/w_1^2)\cos\psi_1, \tag{1a}$$

$$E_{z1} = 2E_{o1}\frac{x_1}{kw_1^2}\exp(-r_1^2/w_1^2)\left(\sin\psi_1 - (z_1/z_R)\cos\psi_1\right), \tag{1b}$$

where E_{o1} is constant, $w_1 = w_o[1 + (z_1/z_R)^2]^{1/2}$ is the laser spot size, w_o is the minimum spot size (waist), $z_R = \pi w_o^2/\lambda$ is the Rayleigh length, $\lambda = 2\pi c/\omega$ is the wavelength, $\omega = ck$ is the frequency, k is the wavenumber, $r_1 = (x_1^2 + y_1^2)^{1/2}$, $\psi = kz_1 - \omega t + \phi_1$, $\phi_1 = r_1^2(z_1/z_R)/w_1^2 - \tan^{-1}(z_1/z_R) + \phi_o$, and ϕ_o is a constant. The longitudinal field component in Eq. (1b) is necessary for the field to be divergence free, i.e., a physically realizable vacuum electromagnetic field. This important field component, among other things, is not considered in Ref. 1. The field components for the second laser beam are given by Eqs. (1a) and (1b) with the subscript 1 replaced by 2. The total transverse, E_x, and axial, E_z, components of the two laser fields in the (x,y,z) coordinate frame are $E_x(x,y,z,t) = (E_{x1} + E_{x2})\cos\theta + (E_{z1} - E_{z2})\sin\theta$, and $E_z(x,y,z,t) = -(E_{x1} - E_{x2})\sin\theta + (E_{z1} +$

$E_{z2})\cos\theta$. In order to have only an axial field component along the z-axis ($x = y = 0$), we set $E_{o1} = E_o$ and $E_{o2} = -E_o$, such that $E_{x1} = -E_{x2}$ and $E_{z1} = E_{z2}$. This gives $E_x(0,0,z,t) = 0$ and

$$E_z(0,0,z,t) = \frac{-2E_o\sin\theta}{(1+\tilde{z}^2\cos^2\theta)^{3/2}}\exp\left(\frac{-(\tilde{z}/\theta_d)^2\sin^2\theta}{1+\tilde{z}^2\cos^2\theta}\right)$$

$$\times\left(\cos\psi + \tilde{z}\cos\theta\sin\psi\right)$$

$$= \frac{-2E_o\sin\theta}{1+\tilde{z}^2\cos^2\theta}\exp\left(\frac{-(\tilde{z}/\theta_d)^2\sin^2\theta}{1+\tilde{z}^2\cos^2\theta}\right)\cos\psi_t, \qquad (2a)$$

where

$$\psi = kz_R\tilde{z}\cos\theta - \omega t + \theta_d^{-2}\frac{(\tilde{z}\cos\theta)^3\tan^2\theta}{(1+\tilde{z}^2\cos^2\theta)} - \tan^{-1}(\tilde{z}\cos\theta) + \phi_o, \qquad (2b)$$

$\psi_t = \psi - \tan^{-1}(\tilde{z}\cos\theta)$, $\tilde{z} = z/z_R$ and $\theta_d = w_o/z_R$ is the diffraction angle. For small intersecting angles, $\theta << 1$, the axial accelerating field in Eq. (2a), as seen by an electron traveling with velocity $v_z \simeq c$, is

$$E_z \simeq \frac{-2E_o\theta}{1+\tilde{z}^2}\exp\left(\frac{-\tilde{z}^2(\theta/\theta_d)^2}{1+\tilde{z}^2}\right)\cos\psi_t, \qquad (3a)$$

where

$$\psi_t \simeq -(\gamma_z\theta_d)^{-2}\tilde{z} - (\theta/\theta_d)^2\tilde{z}/(1+\tilde{z}^2) - 2\tan^{-1}\tilde{z} + \phi_o, \qquad (3b)$$

$t = z/v_z$ has been assumed and $\gamma_z = (1 - v_z^2/c^2)^{-1/2}$. The phase velocity of the accelerating field in Eq. (3a) is greater than c and therefore slips ahead of the electron. From Eq. (3b) we find that, near the focal point, $|z| \lesssim z_R$, the distance required for the electron to phase slip by π is given by

$$z_s \simeq \frac{\gamma_c^2\lambda}{1 + \gamma_c^2/\gamma_z^2}, \tag{4}$$

where $\gamma_c = (\theta^2 + 2\theta_d^2)^{-1/2}$ defines a critical energy. In the low energy limit, $\gamma_z \ll \gamma_c$, the slippage distance, $z_s = \gamma_z^2\lambda \ll z_R$, is much less than a Rayleigh length, while in the high energy limit, $\gamma_z \gg \gamma_c$, the slippage distance, $z_s \simeq \lambda/(\theta^2 + 2\theta_d^2)$, can be comparable to a Rayleigh length. The critical energy $W_c = (\gamma_c - 1)mc^2$ for the parameters in Ref. 1 is $W_c \simeq 4$ MeV and, therefore, in what follows we will consider only the high energy limit. The accelerating field given by Eq. (3) is shown in Fig. 2 for the same parameters used in Fig. 2 of Ref. 1, i.e., $\lambda = 1$ μm, $w_0/\lambda = 4.5$ and $w_0/\lambda = 8.1$. The axial electric field on axis can be written as the gradient of an effective potential, $E_z = - z_R^{-1}\partial U/\partial\tilde{z}$, where

$$U(\tilde{z}) = \frac{4E_0}{k\theta} \exp\left[-\frac{(\theta/\theta_d)^2\tilde{z}^2}{1 + \tilde{z}^2}\right]\sin\left[\phi_0 - \frac{(\theta/\theta_d)^2\tilde{z}}{1 + \tilde{z}^2}\right]. \tag{5}$$

The potential in Eq. (5) is shown in Fig. 3. The change in energy of an electron traveling along the z-axis with velocity $\simeq c$, injected at point z_I and extracted at z_F is $\Delta W(z_I,z_F) = |e|[U(z_F) - U(z_I)]$. For the special case of $z_F = z_0$, $z_I = -z_0$, the energy change is

$$\Delta W = \frac{8|e|E_0}{k\theta} \cos\phi_0 \sin\left[\frac{(\theta/\theta_d)^2\tilde{z}_0}{1 + \tilde{z}_0^2}\right]\exp\left[-\frac{(\theta/\theta_d)^2\tilde{z}_0^2}{1 + \tilde{z}_0^2}\right], \tag{6}$$

where $\tilde{z}_0 = z_0/z_R$. The coefficient in Eq. (6) can be written as $8|e|E_0/(k\theta) = 88P^{1/2}\theta_d/\theta$ MeV, where P is the laser power in TW. Note that as $z_0 \to \infty$, $\Delta W \to 0$, which is a special case of the Lawson-Woodward theorem. A finite energy gain can only occur over a finite interaction range. The maximum energy gain occurs when $2z_0$ is equal to a slippage distance, which is the

distance over which $-E_z > 0$, i.e., the width of the central peak in Fig. 2. For the parameters of Ref. 1, $\lambda = 1$ μm $\theta = 0.1$ rad, and $w_0/\lambda = 4.5$ (8.1), we find an optimal interaction distance of $2z_0 = z_s = 56$ μm (79 μm), $z_R =$ 64 μm (210 μm), and $\theta_d = 71$ mrad (39 mrad). For a finite interaction distance, $2z_0 = z_s$, we find that $\Delta W[\mathrm{MeV}] = 30\ P^{1/2}[\mathrm{TW}]$ for $w_0/\lambda = 4.5$ and $\Delta W[\mathrm{MeV}] = 26\ P^{1/2}[\mathrm{TW}]$ for $w_0/\lambda = 8.1$. Hence, for $P = 20$ TW, $\Delta W \simeq 130$ MeV (110 MeV) for $w_0/\lambda = 4.5$ (8.1), which implies an average acceleration gradient $e\langle E_z\rangle = \Delta W/z_s$ of $\langle E_z\rangle = 2.3$ TV/m (1.4 TV/m).

In Ref. 1 the transformation of the fields and coordinates from the rotated frames to the accelerated frame is in error. The fields in Ref. 1 are not physically realizable, i.e., $\nabla\cdot\underset{\sim}{E} \neq 0$, because the field components E_{z1} and E_{z2} are neglected. In Ref. 1 the electron-laser slippage distance (acceleration distance) is essentially independent of the laser spot size as shown in Fig. 2 of Ref. 1. This is incorrect. Our analysis shows that the slippage distance decreases with decreasing spot size resulting in substantially less energy gain than is indicated in Fig. 3 of Ref. 1. The fact that the net electron energy change vanishes for infinite interaction distances in the crossed beam configuration is consistent with the Lawson-Woodward (L-W) theorem.[2-5]

The L-W theorem[2-5] states that the net energy gain of a relativistic electron interacting with an electromagnetic field in vacuum is zero. The theorem assumes: (i) the laser field is in vacuum with no walls or boundaries present, (ii) the electron is highly relativistic ($v \simeq c$) along the acceleration path, (iii) no static electric or magnetic fields are present, (iv) particle radiative effects are neglected, (v) the region of interaction is infinite, and (vi) ponderomotive effects (nonlinear forces, e.g., the $\underset{\sim}{v} \times \underset{\sim}{B}$ force) are neglected. Under the above assumptions, the

lack of an energy gain can be shown in a straightforward way by considering the general solution to the vacuum wave equation. In order to achieve a nonzero energy gain in vacuum, one or more of the above assumptions must be violated. For example, a finite acceleration region, typically on the order of a few z_R, can be achieved using various optical configurations.[5,6] An important issue in these configurations is the laser intensity damage threshold of optical components. Alternatively, one can rely on nonlinear forces to produce the desired acceleration, such as the ponderomotive force. The use of ponderomotive forces can result in substantial energy gains even for an infinite interaction region. As an example of an accelerator based on the nonlinear ponderomotive force we propose the vacuum beat wave accelerator (VBWA).

In the VBWA two laser beams of different frequencies are co-propagated in the presence of an injected electron beam, see Fig. 4. Properly phased electrons, traveling essentially along the same axis as the two laser beams, experience an axial acceleration from the beat term in the $\underset{\sim}{v} \times \underset{\sim}{B}$ force. Two laser beams of different frequency can be obtained, for example, by splitting a single laser pulse, frequency doubling one of the pulses and recombining the pulses. The acceleration mechanism in the VBWA is similar to that of the inverse free electron laser (IFEL).[7-13] In effect, the wiggler field in the IFEL is replaced by one of the lasers in the VBWA.

In the following analysis of the VBWA, the spot sizes of the laser beams are taken to be large compared to their wavelength and the lasers are assumed to be circularly polarized. The total laser field, represented by the vector potential, is $\underset{\sim}{A}(z,r,t) = \underset{\sim}{A}_1(z,r,t) + \underset{\sim}{A}_2(z,r,t)$ where $\underset{\sim}{A}_1$ and $\underset{\sim}{A}_2$ represent laser 1 and 2 respectively and are given by

$$A_i(z,r,t) = A_{oi}\frac{w_{oi}}{w_i(z)}\exp(-r^2/w_i^2(z))\left[\cos(\psi_i)\hat{e}_x + \sin(\psi_i)\hat{e}_y\right], \quad (7)$$

where the subscript $i = 1,2$ denotes the laser beam, $w_i(z) = w_{oi}(1 + (z/z_{Ri})^2)^{1/2}$, $z_{Ri} = \pi w_{oi}^2/\lambda_i$, $\lambda_i = 2\pi c/\omega_i$, $\omega_i = ck_i$, $\psi_i = k_i z - \omega_i t + \phi_i$, $\phi_i = r^2(z/z_{Ri})/w_i^2(z) - \tan^{-1}(z/z_{Ri}) + \phi_{oi}$, and ϕ_{oi} is constant. In the one-dimensional limit ($\lambda_i/w_{oi} \ll 1$), the axial component of the nonlinear ponderomotive force, F_z, is given by $-|e|(v \times B/c)_z$. In this limit, conservation of transverse canonical momentum implies $v_\perp = ca/\gamma$, where $a = a_1 + a_2$, $a_{1,2} = |e|A_{1,2}/mc^2$ are the normalized vector potentials and γ is the electron relativistic mass factor. Hence, $|e|(v \times B/c)_z = (mc^2/\gamma)[a \times (\nabla \times a)]_z = (mc^2/2\gamma)\partial(a\cdot a)/\partial z$, and the axial component of the pondermotive force is

$$F_z = \frac{-mc^2}{2\gamma}\frac{\partial}{\partial z}(a\cdot a). \quad (8)$$

Substituting Eq. (7) into Eq. (8) we find that

$$F_z \simeq a_{o1}a_{o2}\Delta k\frac{mc^2}{\gamma}\sin(\psi_2 - \psi_1), \quad (9)$$

where $\omega_2 - \omega_1 = \Delta\omega = c\Delta k > 0$ and nonresonant (slowly varying) terms proportional to a_{oi}^2 have been neglected. The effective accelerating gradient is inversely proportional to the electron energy. The phase velocity of the accelerating field on axis and near the focus of the two laser beams, i.e., $r = 0$ and $|z| < z_{R1}, z_{R2}$, is

$$v_{ph} \simeq \frac{\Delta\omega c}{\Delta\omega + c/z_{R1} - c/z_{R2}},$$

$$\simeq c\left[1 - (1 - z_{R1}/z_{R2})/(\Delta k z_{R1})\right], \quad (10)$$

which is less than c for $z_{R2} > z_{R1}$ and can be controlled by appropriately choosing the laser spot sizes and, in addition, by including higher order diffractive effects. The acceleration distance is limited not by the slippage distance but by the diffraction range, i.e., Rayleigh length. For a properly phased electron, the maximum rate of change of energy is

$$\frac{\partial W}{\partial z} = a_{o1} a_{o2} \Delta k mc^2/(1 + W/mc^2), \qquad (11)$$

where we have set $\sin(\psi_2 - \psi_1) = 1$, and $W = mc^2(\gamma - 1)$ is the electron energy. For the purposes of illustration we set $a_o = a_{o1} = a_{o2}$, $z_R = z_{R1} = z_{R2}$ and take the acceleration range to be two Rayleigh lengths, i.e., from $z = -z_R$ to $+z_R$. Equation (11) yields

$$W_F[\mathrm{MeV}] = \left[W_I^2[\mathrm{MeV}] + 480(\lambda_1/\lambda_2 - 1)P_1[\mathrm{TW}]\right]^{1/2}. \qquad (12)$$

where W_F (W_I) is the final (initial) electron energy and we assumed $W_I \gg mc^2$. Since the L-W theorem does not apply to the VBWA mechanism, it can be shown that Eq. (12) is approximately correct even for infinite interaction distances. For circularly polarized radiation, the laser parameters, a_{oi}, λ_i and w_{oi}, are related to the laser power by $P_i[\mathrm{TW}] = 0.043(a_{oi}w_{oi}/\lambda_i)^2$. When the energy change is much greater than the initial energy we obtain $W_F[\mathrm{MeV}] \simeq 22(\lambda_1/\lambda_2 - 1)^{1/2}P_1[\mathrm{TW}]^{1/2}$. As an illustration, for $\lambda_1 = 2\lambda_2 = 1$ µm and $P_1 = 20$ TW, the energy gain is 100 MeV. The energy gain in the VBWA can exceed that in the standard laser wakefield accelerator (LWFA) configuration,[14] which assumes a single laser pulse undergoing vacuum diffraction.

We have shown that the concept of using two crossed laser beams in vacuum to accelerate electrons, as recently discussed by Haaland, yields no net energy gain for highly relativistic electrons and infinite interaction

distances. Reference 1 arrives at incorrect conclusions because important terms contributing to the magnitude and phase of the accelerating electric field were neglected. A relativistic electron can obtain a net energy gain with this configuration if the interaction distance is limited to a few Rayleigh lengths, for example, by reflecting optical components. A major limitation is imposed by the damage threshold of the optical components. We have also proposed a vacuum beat wave accelerator (VBWA) configuration which relies on the nonlinear ponderomotive forces associated with two laser beams of different frequencies. The mechanism behind the VBWA does not satisfy the assumptions of the Lawson-Woodward theorem and can result in substantial electron energy gains in vacuum. The VBWA has the further advantage that, by appropriately choosing the wavelengths, spot sizes, and focal points of the two lasers, the phase velocity v_{ph} of the beat wave can be adjusted. Tuning the phase velocity to a value $v_{ph} < c$ allows for the possibility of accelerating electrons with relatively low injection energies.

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References

1. C.M. Haaland, Optics Comm. 114, 280 (1995).
2. J.D. Lawson, Rutherford Laboratory Report No. RL-75-043 (1975); IEEE Trans. Nucl. Sci. NS-26, 4217 (1979).
3. P.M. Woodward, J. IEE 93, 1554 (1947).
4. R.B. Palmer, Lecture Notes in Physics 296: Frontiers of Particle Beams, edited by M. Month and S. Turner (Springer-Verlag, Berlin, 1988), p. 607; Particle Accel. 11, 81 (1980).
5. E.J. Bochove, G.T. Moore, and M.O. Scully, Phys. Rev. A 46, 6640 (1992).
6. L.C. Steinhauer and W.D. Kimura, J. Appl. Phys. 72, 3237 (1992).
7. A.A. Kolomenskii and A.N. Lebedev, Sov. Phys. JETP 23, 733 (1966).
8. R.B. Palmer, J. Appl. Phys. 43, 3014 (1972).
9. P. Sprangle and Cha-Mei Tang, Nucl. Sci. Vol. NS-28, No. 3, 3346 (1981).
10. R.H. Pantell and T.I. Smith, Appl. Phys. Lett. 40, 753 (1982).
11. E.D. Courant, C. Pellegrini, and W. Zakowicz, Phys. Rev. A 32, 2813 (1985).
12. A.C. Ting and P. Sprangle, Part. Accel. 22, 149 (1987).
13. T.B. Zhang and T.C. Marshall, Phys. Rev. E 50, 1491 (1994).
14. P. Sprangle, E. Esarey, A. Ting, and G. Joyce, Appl. Phys. Lett. 53, 2146 (1988); E. Esarey, P. Sprangle, J. Krall, A. Ting, and G. Joyce, Phys. Fluids B 5, 2690 (1993).

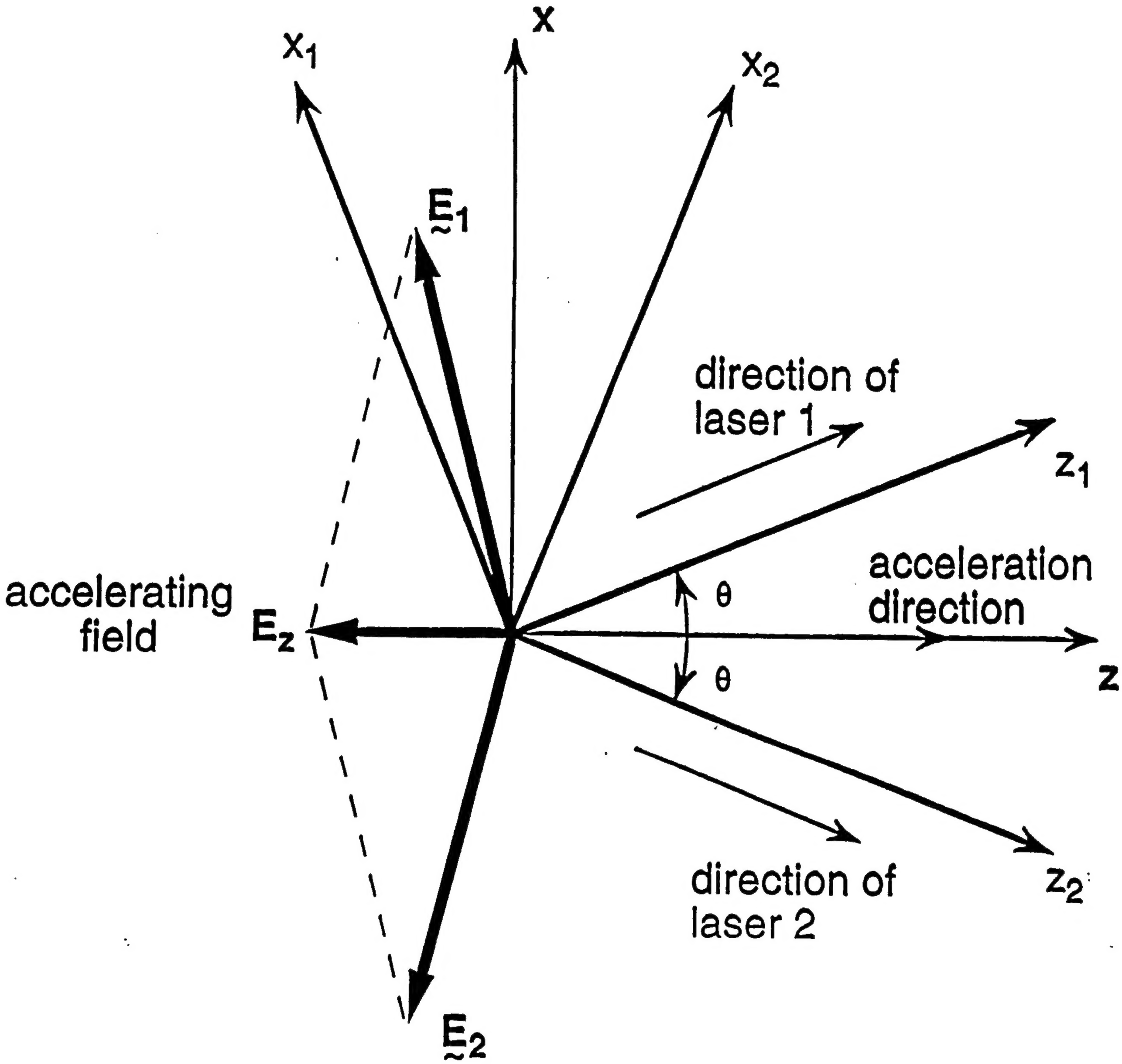

Fig. 1 Coordinate system and electric fields for two intersecting laser beams.

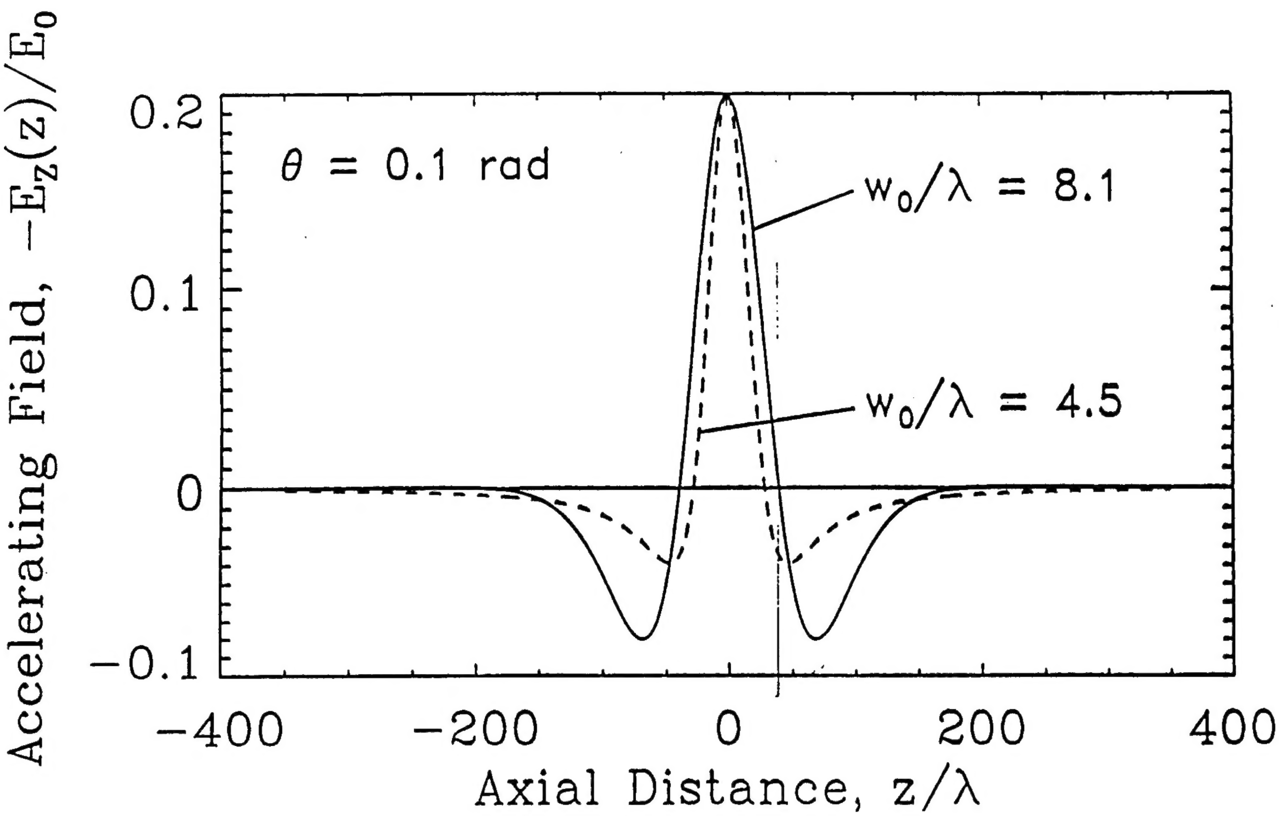

Fig. 2 Accelerating axial field $-E_z$ plotted versus position along the z-axis.

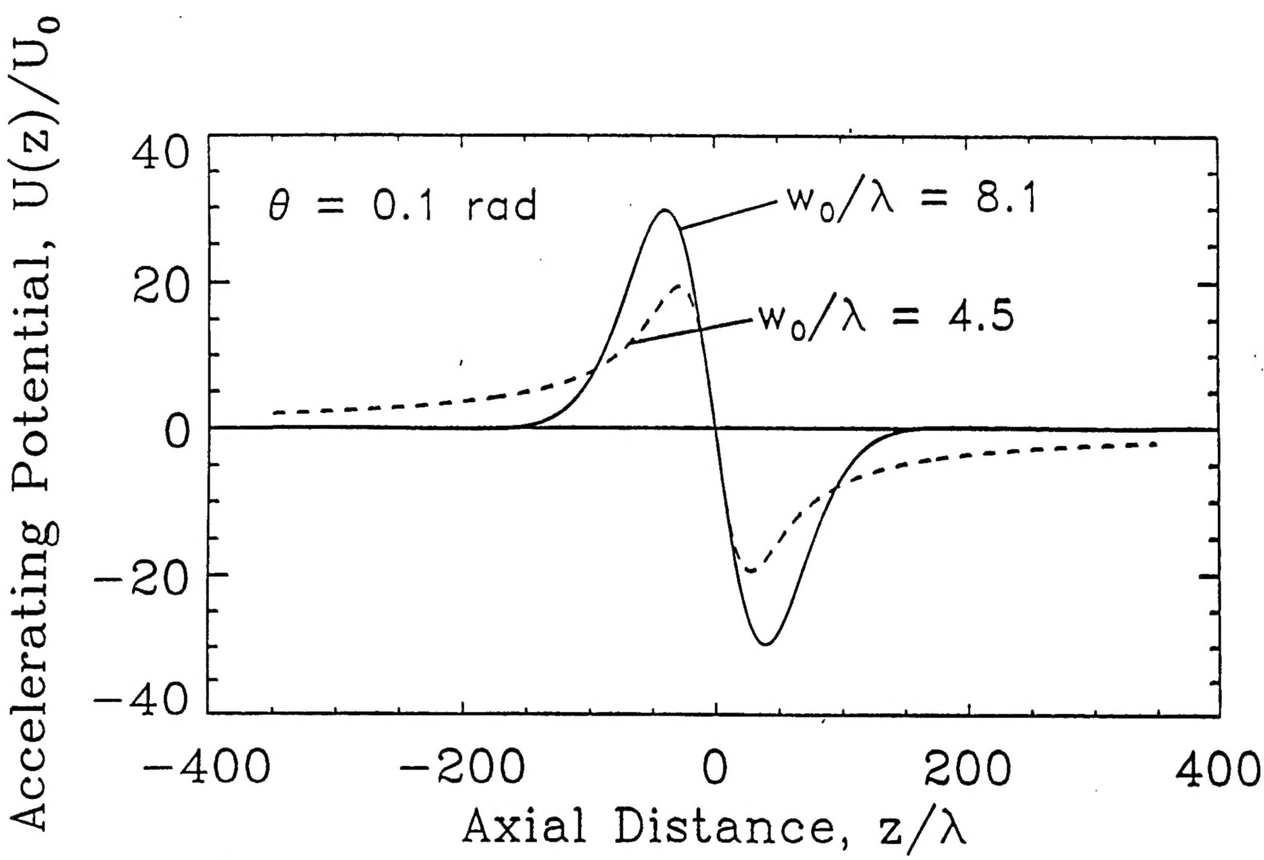

Fig. 3 Effective potential U(z) plotted versus position along the z-axis.

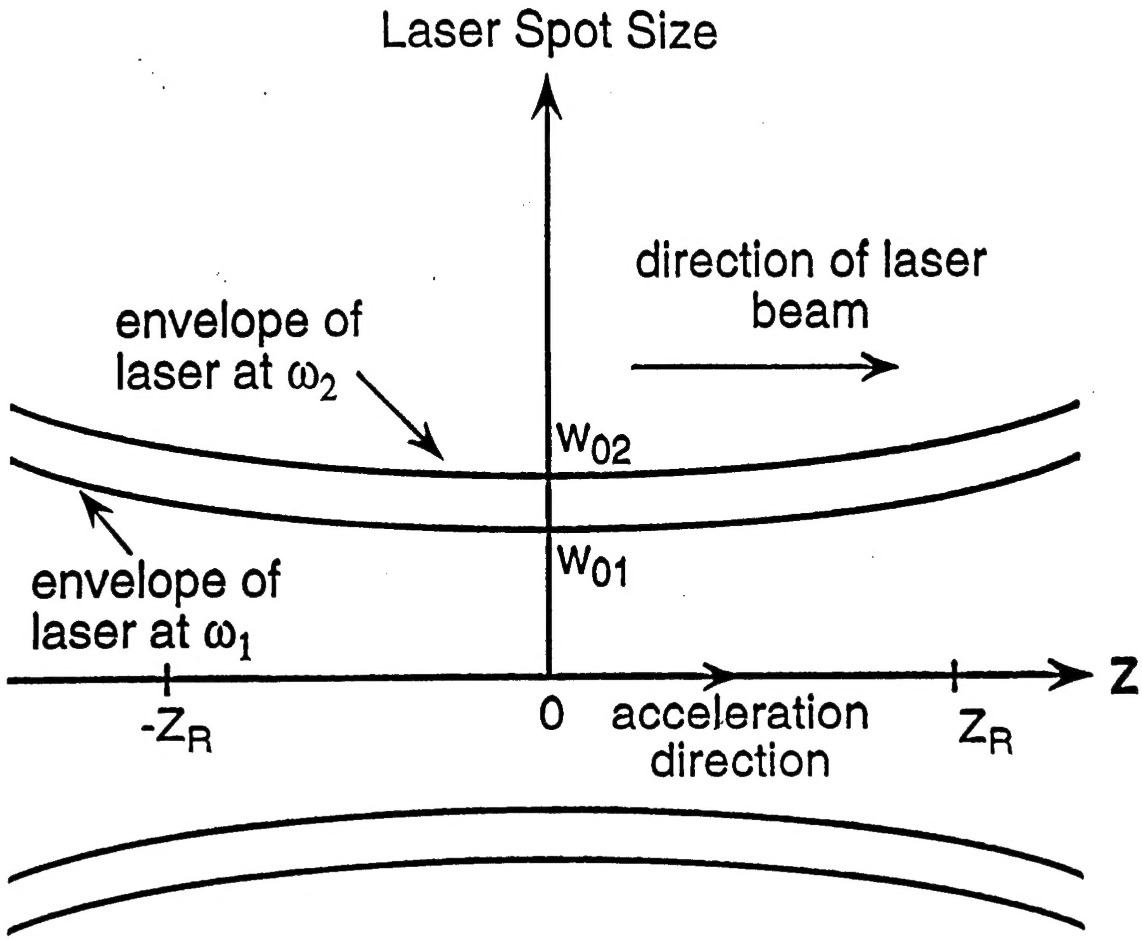

Fig. 4 The VBWA configuration.